Computation of Two-Dimensional, Viscous Nozzle Flow

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Theme

THE calculation of viscous nozzle flows can be accomplished by either solving the inviscid-core and viscous-boundary-layer equations separately or by solving the viscous equations for the entire flowfield. In the inviscid-core, boundary-layer approach, the assumption is made that the boundary layer is thin when compared to the nozzle diameter. However, for Reynolds numbers on the order of 10^3 based on the throat diameter, this assumption is questionable. On the other hand, while the viscous equation approach is physically desirable, the computations tend to be rather lengthy. Therefore, the object of this research was to modify an efficient inviscid code, discussed in Refs. 1 and 2, to solve the viscous equations. This new code, called VNAP (Viscous Nozzle Analysis Program), is then used to calculate the flow in a chemical laser nozzle. This numerical solution is compared with both the inviscid-core, boundary-layer solution and experimental data presented in Ref. 3.

Contents

The nonconservative form of the Navier-Stokes equations for two-dimensional, time-dependent flow of a perfect gas is solved. The physical plane is mapped into a rectangular computational plane. The interior mesh points are computed using the MacCormack[4] scheme. The inlet and exit mesh points for subsonic flow are calculated using a reference-plane characteristic scheme where the viscous terms are treated as source functions. For supersonic flow, the inlet mesh point dependent variables are specified and the exit mesh point variables are extrapolated. The wall and centerbody mesh point velocity components are set equal to zero, the temperature is specified, and the density is calculated using the conservation of mass equation. In order to stabilize the solution for shock computations, a local artificial viscosity model is employed.

The nozzle geometry for the results presented here is shown in Fig. 1 and is configuration 1 ($5 \times$ size) of Ref. 3. Figure 2 gives the steady-state lines of constant Mach number for a throat Reynolds number of 1200 based on the throat gap. The bottom line of the frame is the nozzle mid-plane and the flow is from the left to the right. The lowest and highest contour lines are labeled with L and H, respectively. The static wall temperature was set equal to the stagnation temperature. The first coefficient of viscosity μ and thermal conductivity k were assumed to vary as the square root of the temperature. The second coefficient of viscosity λ was set equal to -0.67μ. From Fig. 2, it is seen that the boundary layer grows very rapidly in the supersonic portion of the nozzle. Figure 3 gives the velocity profile at $x = 1.65$ cm (0.65 in.). The quantity M^* is the velocity magnitude divided by the speed of sound at the nozzle throat assuming one-dimensional, isentropic flow, and Y_w is the height of the nozzle wall at $x = 1.65$ cm. The experimental data are those of Ref. 3. From Fig. 3, it is seen that the present theory agrees well with the experimental data, as does the inviscid-core, boundary-layer theory of Ref. 3 (not shown). However, the pressure in the boundary layer just downstream of the throat varied as much as 20% in the radial direction, thereby, making the constant radial pressure assumption of a boundary-layer technique somewhat questionable. This calculation employed an 80×21 uniform mesh and required approximately 1000 time steps to reach steady state. The computational time was 6 min on a CDC-7600 computer. Figure 4 shows the discharge coefficient for throat Reynolds numbers of 600 to 3600. From Fig. 4, we note that the present solution is superior to the inviscid-core, boun-

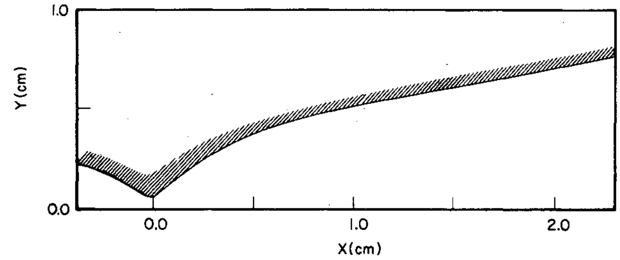

Fig. 1 Rocketdyne configuration No. 1 geometry.

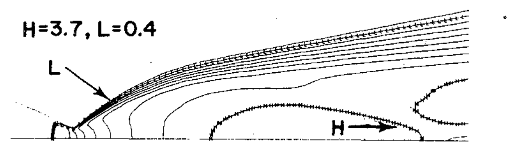

Fig. 2 Lines of constant Mach number for $Re^* = 1200$.

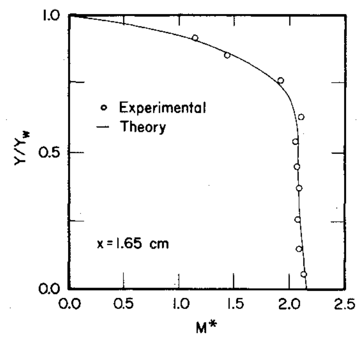

Fig. 3 Velocity profile at $x = 1.65$ cm for $Re^* = 1200$.

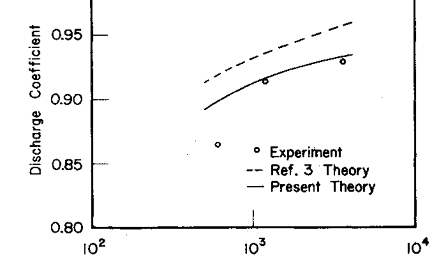

Fig. 4 Discharge coefficient vs Reynolds number.

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Index categories: Subsonic and Transonic Flow; Supersonic and Hypersonic Flow; Nozzle and Channel Flow.

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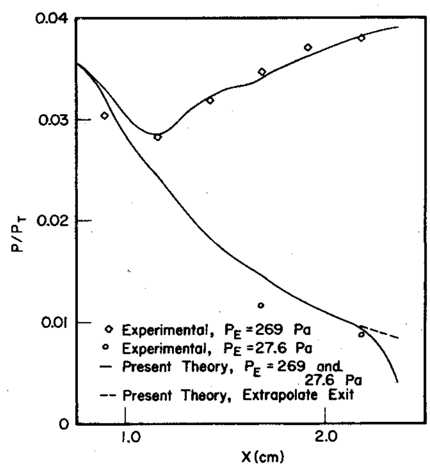

Fig. 5 Wall pressure ratio for $Re^* = 1200$.

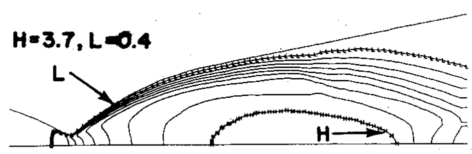

Fig. 6 Lines of constant Mach number for the separated case.

dary-layer solution of Ref. 3. The authors of Ref. 3 had some reservations concerning the accuracy of the $Re^* = 600$ experimental data point and theoretical solution.

For these calculations, values for the exit column of mesh points were extrapolated from those of the interior mesh points. This resulted for $Re^* = 1200$ in a wall pressure at the nozzle exit of 60.8 Pa (0.0088 psia). To determine the sensitivity of this flow to the downstream plenum pressure, a different nozzle exit boundary condition was applied. The static pressure for all exit mesh points in the subsonic region was specified, i.e. the characteristic scheme for subsonic outflow was employed. Figure 5 shows the wall pressure for downstream plenum pressure of 27.6 Pa (0.004 psia) and 269.0 Pa (0.039 psia). From Fig. 5, it is seen that the 27.6 Pa plenum pressure produced no significant changes in the nozzle flow when compared with the extrapolated case. In fact, the expansion from the higher wall pressure to the specified exit pressure occurred over only 4 mesh lengths in the x-direction and two in the y-direction. Reference 3 did not present any theoretical wall pressure results. On the other hand, the 269.0 Pa plenum pressure caused the boundary layer to separate from the nozzle wall. From Fig. 5, it is seen that there is reasonably good agreement with the experimental data of Ref. 3. The inviscid-core, boundary-layer technique of Ref. 3 is not able to calculate separated flows. Figure 6 shows the steady-state lines of constant Mach number for the separated case.

References

[1] Cline, M.C., "Computation of Steady Nozzle Flow by a Time-Dependent Method," *AIAA Journal*, Vol. 12, April 1974, pp. 419-420.

[2] Cline, M.C., "NAP: A Computer Program for the Computation of Two-Dimensional, Time-Dependent, Inviscid Nozzle Flow" Los Alamos Scientific Laboratory Report LA-5984, Los Alamos, New Mex. (to be published).

[3] Hyde, J.C., Hosack, G.A., and Osugi, G.S., "An Investigation of Velocity Flowfields in Chemical Laser Nozzles," AIAA Paper 73-641, Palm Springs, Calif., 1973.

[4] MacCormack, R.W., "The Effect of Viscosity in Hypervelocity Impact Cratering," AIAA Paper 69-354, Cincinnati, Ohio, 1969.